Integrating Slacks-based Measure of Efficiency and Super-efficiency in Data Envelopment Analysis

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Abstract

In this paper, we develop an integrated model for slacks-based measure (SBM) simultaneously of both the efficiency and the super-efficiency for decision-making units (DMUs) in data envelopment analysis (DEA). Unlike the traditional solution approaches in which we need to identify the efficient DMUs by the SBM model of Tone [20] before applying the super SBM model of Tone [21] for the DMUs to achieve their super-efficiency scores, our integration can obtain the efficiency scores of the inefficient DMUs and the super-efficiency scores of the efficient DMUs by solving simultaneously these two models by an one-stage approach. Therefore, it may save computational time for large-scale practical applications. Due to the non-linearity in the objective function of this integrated model, we develop a linearisation technique to deal with the non-linear model. The numerical experiments, carried out on several examples in the literature and a case study, have demonstrated the accuracy and the computational time effectiveness of our proposed model as compared with the traditional solution approaches.

Keywords: data envelopment analysis (DEA); slacks-based measure; efficiency; super-efficiency; one-stage approach; linearisation.

1 Introduction

Data envelopment analysis (DEA) is a methodology in operations research and economics for performance evaluation and benchmarking, considering multiple performance measures. It is useful for empirically measuring the productive efficiency of decision-making units (DMUs), for example, organisations, banks, etc. Since the first publication of measuring the efficiency of DMUs proposed by Charnes et al. [4], there has been a continuous and recently rapid growth in the field of DEA in terms of both practical application and theory. As for the perspective of practical application for the first 20 years of DEA development, the top five application fields include banking, healthcare, agriculture and farming, transportation, and education

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[15]. In the recent years, while banking, agriculture and farming, and transportation have still been in the top five application fields of DEA, supply chain and public policy have appeared as two emergent application fields of DEA. In addition, some novel DEA applications include the corporate management of securities [23], the automotives [19], tourism in the Coral Triangle region [13], the thermal power generation [18], etc. In addition to the appearance of many novel practical applications, the total number of journal articles in DEA reached 10,300 with 11,975 individuals. This demonstrates the increasingly important role of DEA applications in both public and private sectors. Emrouznejad and Yang [9] provide a comprehensive survey and analysis of the first 40 years of DEA related studies.

In DEA, since the efficiency of a DMU is defined as the ratio of multiple inputs and outputs, the objective of DMU is the utilisation of minimum inputs to produce maximum outputs. Obviously, a DMU is known to be more efficient than other DMU if it uses the same amount of inputs to produce more outputs, or a lesser amount of inputs to produce the same outputs. In the literature, there are two approaches to evaluate the performance of DMUs, i.e., the measure of efficiency and the measure of super-efficiency. Both approaches can distinguish the sets of inefficient DMUs and efficient DMUs. However, the former approach only gauges the scores of inefficient DMUs (i.e., values range from 0 to 1), while the latter approach only gauges the scores of efficient DMUs (i.e., values are greater than 1).

As for the perspective of theory in the measure of efficiency, several DEA models have been constructed to overcome the shortcomings of the first DEA model [4]. In the first model, a DMU with the efficiency score equal to one might be inefficient since it could not account for all efficiency components of a DMU [16] (known as radial efficiency measure). Banker et al. [3] proposed an input-oriented model to evaluate the efficiency of a DMU by solving a linear program with a new separate variable which is the dual variable associated with the constraint of *returns to scale*. It is possible to determine whether operations are conducted in regions of increasing, constant or decreasing *returns to scale* in the multiple input and multiple output situations. While the above-mentioned models require to distinguish between input-oriented and output-oriented objective functions, Charnes et al. [5] developed an additive model to measure the efficiency of a DMU based on considering the total slacks of inputs and outputs simultaneously in arriving at a point on the efficient frontier that are constructed by a set of efficient DMUs. The additive model can account for all inefficiency components of a DMU that the previous models could not. Therefore, if a DMU possesses zero slacks, it is efficient. However, this additive model does not provide directly an efficiency measure in the objective function. Tone [20] augmented the additive model by introducing a slacks-based measure (SBM), in which the slack variables represent excesses in inputs and shortfalls in outputs, to identify directly the efficiency score of a DMU in the objective function. In the SBM model, a DMU with efficiency score equal to one is strongly efficient (known as a representative of non-radial efficiency measures). For a survey of methodological development of the various models for measuring efficiency, readers can refer to [10] and [7].

As for the perspective of theory in the measure of super-efficiency, Andersen and Petersen [1] proposed a radial super-efficiency model to measure the scores of the efficient DMUs while remaining unchanged the scores of the inefficient DMUs. This model can differentiate the efficient DMUs that the traditional DEA models above-mentioned can not. However, such the super-efficiency model is mainly applicable for constant *returns to scale* (CRS) since it may be infeasible as variable *returns to scale* (VRS) is used [17, 6, 14]. Unlike the model of Andersen and Petersen [1] based on the radial super-efficiency measure approach, Tone [21] developed a super SBM model with non-radial super-efficiency measure (i.e., dealing with input/output slacks directly) to differentiate the efficient DMUs. This model is useful if the number of DMUs is small as compared with the number of evaluation criteria. Fang et al. [11] constructed a two-stage solution approach for determining the super-efficiency scores of the efficient DMUs and the efficiency scores of the inefficient DMUs. In the approach, the super SBM model is solved first and then the SBM model is applied. The authors show that the results obtained can bring a stronger Pareto efficient projection than the super SBM model, while the efficiency scores of DMUs remain unchanged as compared with those of [20] and [21].

Du et al. [8] extended the super SBM model of Tone [21] to the additive (slacks-based) DEA model. Unlike the traditional radial super-efficiency DEA models, this model is always feasible under VRS condition. As a result, a complete ranking of the efficient DMUs can be obtained. However, the authors use different slacks-based objective functions in their model. Thus, a post-computation process is required to obtain the efficiency scores of DMUs. In addition, the model requires the set of efficient DMUs to be determined before applying the additive super-efficiency model to measure the efficiency scores of the DMUs, which may be overly time-consuming in the implementation of large-scale practical applications. Therefore, Guo et al. [12] have recently proposed an integration of the additive (slacks-based) DEA models for determining the efficiency scores of the inefficient DMUs and the super-efficiency scores of the efficient DMUs by solving an one-stage model. The one-stage solution approach can save computational time for large-scale practical applications, for example, computing the SBM-based Malmquist productivity index used to evaluate the efficiency change over time [22]. In addition, the projections identified by the model are strongly efficient. However, like the model of Du et al. [8], the integrated model requires potentially time-consuming post-computation to obtain the efficiency scores of DMUs. Table 1 summaries the development of the above-mentioned DEA models with their properties. In the table, we can see that the one-stage solution approach based on the SBM and super SBM models with an objective function that can directly measure the efficiency and super-efficiency scores of DMUs has not been studied.

We have developed an integration of the SBM [20] and super SBM [21] models to be able to directly obtain the efficiency scores of the inefficient DMUs and the super-efficiency scores of the efficient DMUs by solving one-stage model (see Figure 1). Like the integrated model of Guo et al. [12], our model may save computational time for large-scale practical applications. In addition, since our objective function can directly determine the efficiency and super-efficiency scores of DMUs, it does not need a post-computation process as the model of Guo et al. [12]. This may save much computational time in the applications with the large number of DMUs. Due to the non-linearity in the objective function of our integrated model, a linearisation technique is developed to deal with the non-linear model. The linearisation technique can be easily applied for similar models in other fields. To overcome the negative or zero cases of observed input and output values in the practical applications, we propose a strategy to scale all the original input and output values. The linearised model with the scaling strategy may obtain the robustness of the relative efficiency measure for DMUs. Besides that numerical experiments are carried out on several examples in the literature, we evaluate and compare our model with other models in a case study with the large number of DMUs, inputs and outputs.

The main contribution in this paper is (i) a novel one-stage solution approach based on the SBM and super SBM models, (ii) a direct objective function to obtain the efficiency and super-efficiency scores of DMUs without the post-computation process, (iii) a linearisation technique to deal with the non-linear integrated model, (iv) a scaling strategy to handle the negative or zero cases of inputs and outputs in the real-world applications, and (v) a large-size case study to demonstrate the performance of our model. The remaining of this paper is organised as follows. Section 2 reviews the SBM model [20] and the super SBM model [21]. Section 3 presents how to integrate these two models into an one-stage model for simultaneously measuring the efficiency scores of both the inefficient and efficient DMUs. In this section, we also introduce the linearisation technique to deal with the non-linear integrated model, and the scaling strategy. Section 4 is the results of

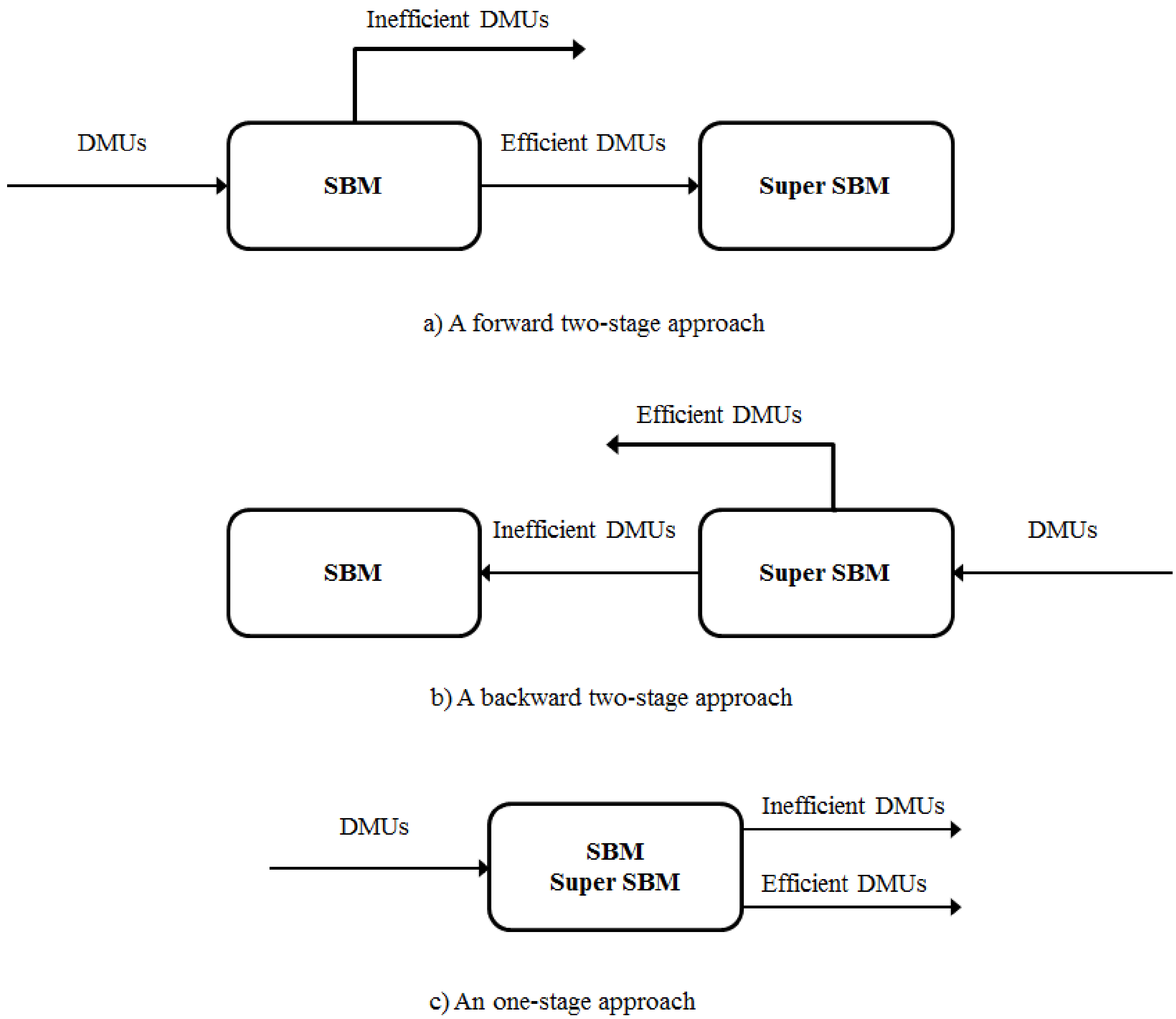

Figure 1: An illustration of the forward and backward two-stage approaches vs. the one-stage approach.

Table 1: A summary of efficiency and super-efficiency slacks-based measure development in DEA.

Paper	Efficiency measure		Additive DEA	Stages		Solution approach	
	Radial	Non-radial		Efficiency	Super-efficiency	Two-stage	One-stage
Charnes et al. [4]	x			x			
Banker et al. [3]	x			x			
Charnes et al. [5]	x		x	x			
Andersen and Petersen [1]	x		x		x		
Tone [20]		x		x			
Tone [21]		x			x		
Du et al. [8]		x	x		x		
Fang et al. [11]		x		x	x	x	
Guo et al. [12]		x	x	x	x		x

numerical experiments to illustrate the accuracy and the computational time effectiveness of our proposed model. Finally, conclusions and future work are provided in Section 5.

2 Slacks-based Measure of Efficiency and Super-efficiency

The SBM model and the super SBM model proposed by Tone [20] and Tone [21] are reviewed, respectively. These models are then integrated into our one-stage model in the next section.

2.1 Slacks-based measure of efficiency

Assume that we deal with a set of n DMUs in which each has m inputs and s outputs. We denote the $i$the input and the rth output of DMU$_j$ by x_{ij} $(i = 1, .., m;\ j = 1, .., n)$ and y_{rj} $(r = 1, .., s;\ j = 1, .., n)$, respectively. Then, based on the SBM model of Tone [20], the efficiency score of the target DMU$_k$ is evaluated by

[SBM]:

$$\min\ \rho_k = \frac{1 - \dfrac{1}{m}\displaystyle\sum_{i=1}^{m} \frac{s_i^-}{x_{ik}}}{1 + \dfrac{1}{s}\displaystyle\sum_{r=1}^{s} \frac{s_r^+}{y_{rk}}}, \tag{1}$$

$$\text{s.t.: } x_{ik} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^-,\ i = 1, .., m, \tag{2}$$

$$y_{rk} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+,\ r = 1, .., s, \tag{3}$$

$$\lambda_j \geq 0,\ j = 1, .., n, \tag{4}$$

$$s_i^- \geq 0,\ i = 1, .., m, \tag{5}$$

$$s_r^+ \geq 0,\ r = 1, .., s, \tag{6}$$

where s_i^- $(i = 1, .., m)$ and s_r^+ $(r = 1, .., s)$ are slacks representing input excess and output shortfall, respectively; and λ is a non-negative vector.

In this model, all the data of inputs and outputs are assumed to be positive, i.e., $x_{ij} > 0$ and $y_{rj} > 0$ $(i = 1, .., m;\ r = 1, .., s;\ j = 1, .., n)$, due to the objective function. The objective value is less than or equal to 1. We obtain $\rho_k^* < 1$ for the inefficient DMUs and $\rho_k^* = 1$ for the efficient DMUs as solving the SBM model.

2.2 Slacks-based measure of super-efficiency

After solving the SBM model to obtain the set of efficient DMUs (i.e., $\rho^* = 1$), the super SBM model proposed by Tone [21] is applied to evaluate the efficient DMUs. For an efficient DMU$_k$, we solve the following problem to identify its super-efficiency score.

[SupSBM]:

$$\min\ \delta_k = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{\tilde{y}_r}{y_{rk}}}, \tag{7}$$

$$\text{s.t.: } \tilde{x}_i \geq \sum_{j=1, j\neq k}^{n} x_{ij}\lambda_j,\ i = 1, .., m, \tag{8}$$

$$\tilde{y}_r \leq \sum_{j=1, j\neq k}^{n} y_{rj}\lambda_j,\ r = 1, .., s, \tag{9}$$

$$\tilde{x}_i \geq x_{ik},\ i = 1, .., m, \tag{10}$$

$$0 \leq \tilde{y}_r \leq y_{rk},\ r = 1, .., s, \tag{11}$$

$$\lambda_j \geq 0,\ j = 1, .., n,\ j \neq k, \tag{12}$$

where $\tilde{x}_i$ $(i = 1, .., m)$ and $\tilde{y}_r$ $(r = 1, .., s)$ are decision variables with respect to inputs and outputs, respectively; while other parameters are defined as in the last section. Note that the super SBM model is the same as Tone [21], but is expressed by different notations, which makes identical with our one-stage model in next section.

As solving the SupSBM model for the efficient DMUs pre-identified, we obtain their super-efficiency scores $\delta_k^* > 1$. Then, the efficiency and super-efficiency scores of all the DMUs are determined. All these scores can also be found by solving the SupSBM model first and then applying the SBM model for the inefficient DMUs (i.e., $\delta_k^* = 1$), known as the reversed (or backward) two-stage solution approach proposed by Fang et al. [11].

3 An Integration of the SBM Model and the Super SBM Model

Since our model is integrated based on the above-mentioned SBM model and the super SBM model, it inherits the properties of both models. Our projection results are similar to those of these models. In this paper, we thus do not discuss the issues, but concentrate how to build an one-stage model from these models, and how to solve the model efficiently for practical applications.

3.1 An integrated model

In the section, we develop one-stage model to measure the efficiency and super-efficiency scores of the inefficient and efficient DMUs simultaneously. Our model is based on the integration of the SBM model [20] and the super SBM model [21]. After linearising the SBM and the super SBM models as shown in [20] and [21], respectively, we integrate them into one-stage model. For any DMU$_k$, its efficiency or super-efficiency score can be evaluated by

[OneSupSBM]:

$$\min\ \theta_k = \alpha \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}} + (1-\alpha)\left(t_1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}\right), \tag{13}$$

$$\text{s.t.: } \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}} - 1 \leq \alpha\overline{M}, \tag{14}$$

$$\alpha \in \{0;1\}, \tag{15}$$

$$1 = t_1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{rk}}, \tag{16}$$

$$t_1 x_{ik} = \sum_{j=1}^{n} x_{ij}\lambda_{1j} + s_i^-,\ i = 1,..,m, \tag{17}$$

$$t_1 y_{rk} = \sum_{j=1}^{n} y_{rj}\lambda_{1j} - s_r^+,\ r = 1,..,s, \tag{18}$$

$$\lambda_{1j} \geq 0\ (j = 1,..,n),\ s_i^- \geq 0\ (i = 1,..,m),\ s_r^+ \geq 0\ (r = 1,..,s),\ t_1 > 0, \tag{19}$$

$$1 = \frac{1}{s}\sum_{r=1}^{s}\frac{\tilde{y}_r}{y_{rk}}, \tag{20}$$

$$\tilde{x}_i \geq \sum_{j=1, j\neq k}^{n} \lambda_{2j}x_{ij},\ i = 1,..,m, \tag{21}$$

$$\tilde{y}_r \leq \sum_{j=1, j\neq k}^{n} \lambda_{2j}y_{rj},\ r = 1,..,s, \tag{22}$$

$$\tilde{x}_i \geq t_2 x_{ik},\ i = 1,..,m, \tag{23}$$

$$0 \leq \tilde{y}_r \leq t_2 y_{rk},\ r = 1,..,s, \tag{24}$$

$$\lambda_{2j} \geq 0\ (j = 1,..,n),\ t_2 > 0, \tag{25}$$

where $\overline{M}$ is a big positive number; λ_{1j} and λ_{2j} $(j = 1,..,n)$ represent the non-negative vectors of the SBM model and the super SBM model, respectively; and t_1 and t_2 are two auxiliary variables for linearisation.

The objective function (13) is to measure the super-efficiency score of an efficient DMU (i.e., $\frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}}$) or the efficiency score of an inefficient DMU (i.e., $t_1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}$). In the objective function, we use a binary variable $\alpha \in \{0, 1\}$ to switch the measure of efficiency based on the SBM model or the super SBM model. If $\alpha = 1$, then the super SBM model is chosen to compute the super-efficiency score of DMU_k. If $\alpha = 0$, then the SBM model is chosen to compute the efficiency score of DMU_k. Constraints (14)-(15) are used to control switching between the SBM model and the super SBM model. Constraints (16)-(19) are the constraints of the linearised SBM model, while constraints (20)-(25) are the constraints of the linearised super SBM model.

Next, we explain why the one-stage model is able to switch automatically the SBM model and the super SBM model based on the choice of value α. Let $\theta_{1k} = \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}}$ and $\theta_{2k} = t_1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}$, note that $\theta_{1k} \geq 1$ and $\theta_{2k} \leq 1$.

- **Case 1:** if DMU_k is efficient, our integrated model has to switch into the super SBM model (i.e., $\alpha = 1$). We can prove this as follows. Due to DMU_k is efficient, it has $\theta_{1k}^* > 1$ and $\theta_{2k}^* = 1$. Since $\theta_{1k}^* > 1$, it leads to $\frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}} - 1 > 0$. Constraint (14) becomes $0 < \alpha M$. Then, $\alpha = 1$ to satisfy the constraint. The objective function becomes min $\theta_k = \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}}$. In other words, the super SBM model is selected and only constraints (20)-(25) are active to the objective function. Therefore, we can obtain as the same super-efficiency score as the model of Tone [21].

- **Case 2:** if DMU_k is inefficient, our integrated model has to switch into the SBM model (i.e., $\alpha = 0$). We can prove this as follows. Due to DMU_k is inefficient, it has $\theta^*_{1k} = 1$ and $\theta^*_{2k} < 1$. Since $\theta^*_{1k} = 1$, it leads to $\frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}} - 1 = 0$. Constraint (14) becomes $0 \leq \alpha M$. The value α may be 0 or 1. Since the objective function is minimisation, the model chooses $\alpha = 0$ to obtain the smaller part of the objective value ($\theta^*_{1k} = 1$ vs. $\theta^*_{2k} < 1$). Then , the objective function becomes min $\theta_k = t_1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}$. In other words, the SBM model is selected and only constraints (16)-(19) are active to the objective function. Therefore, we can obtain as the same efficiency score as the model of Tone [20].

3.2 A linearised model

The objective function is a non-linear function. To be able to solve this problem, we need to develop a linearisation technique to linearise the non-linear terms (i.e., αs_i^-, $\alpha\tilde{x}_i$ and αt_1) of the objective function. Let $u_i = \alpha s_i^-$ (where $0 \leq s_i^- \leq t_1 x_{ik}$), $v_i = \alpha\tilde{x}_i$ (where $\tilde{x}_i \geq x_{ik}$) and $w = \alpha t_1$ (where $0 < t_1 \leq 1$), then replace the non-linear terms of the objective function by the new variables. We can linearise the objective function (13) by

$$\min\ \theta_k = \frac{1}{m}\sum_{i=1}^{m}\frac{u_i}{x_{ik}} + \frac{1}{m}\sum_{i=1}^{m}\frac{v_i}{x_{ik}} - w + t_1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}, \tag{26}$$

$$\text{s.t.: } u_i \leq \alpha t_1 x_{ik},\ i = 1,..,m, \tag{27}$$

$$u_i \leq s_i^-,\ i = 1,..,m, \tag{28}$$

$$u_i \geq s_i^- - (1-\alpha)t_1 x_{ik},\ i = 1,..,m, \tag{29}$$

$$u_i \geq 0,\ i = 1,..,m, \tag{30}$$

$$x_{ik}\alpha \leq v_i \leq \overline{M}\alpha,\ i = 1,..,m, \tag{31}$$

$$\tilde{x}_i - (1-\alpha)\overline{M} \leq v_i \leq \tilde{x}_i - (1-\alpha)x_{ik},\ i = 1,..,m, \tag{32}$$

$$\underline{M}\alpha \leq w \leq \alpha, \tag{33}$$

$$t_1 - (1-\alpha) \leq w \leq t_1 - (1-\alpha)\underline{M}, \tag{34}$$

where $\underline{M}$ and $\overline{M}$ are the small and big positive numbers, respectively.

Constraints (27)-(30) are used to linearise the non-linear term αs_i^-, constraints (31)-(32) are used to linearise the non-linear term $\alpha\tilde{x}_i$, and constraints (33)-(34) are used to linearise the non-linear term αt_1.

Next, we explain why constraints (27)-(30) can linearise αs_i^-. Similar explanations can be applied for the other non-linear terms.

- **Case 1:** if $\alpha = 0$, then $\alpha s_i^- = 0$. We need to prove that constraints (27)-(30) can lead the same result, i.e., $u_i = 0$. We can see that if $\alpha = 0$, then constraint (27): $u_i \leq 0$. From constraint (30): $u_i \geq 0$, we obtain $u_i = 0$. Constraints (28)-(29) satisfy with $u_i = 0$ since $0 \leq s_i^- \leq t_1 x_{ik}$.
- **Case 2:** if $\alpha = 1$, then $\alpha s_i^- = s_i^-$. We need to prove that constraints (27)-(30) can lead the same result, i.e., $u_i = s_i^-$. We can see that if $\alpha = 1$, then constraint (28): $u_i \leq s_i^-$ and constraint (29): $u_i \geq s_i^-$ lead to $u_i = s_i^-$. Constraints (27): $u_i \leq t_1 x_{ik}$ and constraint (30): $u_i \geq 0$ are satisfied since $0 \leq s_i^- \leq t_1 x_{ik}$.

Note that by replacing $w = \alpha t_1$ in constraints (27) and (29), we obtain the full mixed-integer linear programming (MILP) formulation of our one-stage model for measuring the efficiency scores of the efficient DMUs and the super-efficiency scores of the inefficient DMUs as follows.

[OneSupSBM-LP]:

$$\min\ \theta_k = \frac{1}{m}\sum_{i=1}^{m}\frac{u_i}{x_{ik}} + \frac{1}{m}\sum_{i=1}^{m}\frac{v_i}{x_{ik}} - w + t_1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}, \tag{35}$$

$$\text{s.t.: } \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_i}{x_{ik}} - 1 \leq \alpha\overline{M}, \tag{36}$$

$$\alpha \in \{0;1\}, \tag{37}$$

$$1 = t_1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{rk}}, \tag{38}$$

$$t_1 x_{ik} = \sum_{j=1}^{n} x_{ij}\lambda_{1j} + s_i^-,\ i = 1,..,m, \tag{39}$$

$$t_1 y_{rk} = \sum_{j=1}^{n} y_{rj}\lambda_{1j} - s_r^+,\ r = 1,..,s, \tag{40}$$

$$\lambda_{1j} \geq 0\ (j = 1,..,n),\ s_i^- \geq 0\ (i = 1,..,m),\ s_r^+ \geq 0\ (r = 1,..,s),\ t_1 > 0, \tag{41}$$

$$1 = \frac{1}{s}\sum_{r=1}^{s}\frac{\tilde{y}_r}{y_{rk}}, \tag{42}$$

$$\tilde{x}_i \geq \sum_{j=1, j\neq k}^{n} \lambda_{2j}x_{ij},\ i = 1,..,m, \tag{43}$$

$$\tilde{y}_r \leq \sum_{j=1, j\neq k}^{n} \lambda_{2j}y_{rj},\ r = 1,..,s, \tag{44}$$

$$\tilde{x}_i \geq t_2 x_{ik},\ i = 1,..,m, \tag{45}$$

$$0 \leq \tilde{y}_r \leq t_2 y_{rk},\ r = 1,..,s, \tag{46}$$

$$\lambda_{2j} \geq 0\ (j = 1,..,n),\ t_2 > 0, \tag{47}$$

$$u_i \leq w x_{ik},\ i = 1,..,m, \tag{48}$$

$$u_i \leq s_i^-,\ i = 1,..,m, \tag{49}$$

$$u_i \geq s_i^- - (t_1 - w)x_{ik},\ i = 1,..,m, \tag{50}$$

$$u_i \geq 0,\ i = 1,..,m, \tag{51}$$

$$x_{ik}\alpha \leq v_i \leq \overline{M}\alpha,\ i = 1,..,m, \tag{52}$$

$$\tilde{x}_i - (1-\alpha)\overline{M} \leq v_i \leq \tilde{x}_i - (1-\alpha)x_{ik},\ i = 1,..,m, \tag{53}$$

$$\underline{M}\alpha \leq w \leq \alpha, \tag{54}$$

$$t_1 - (1-\alpha) \leq w \leq t_1 - (1-\alpha)\underline{M}. \tag{55}$$

Then, it is solvable by any commercial MILP solver.

In addition, as discussed in Banker and Chang [2] the procedure of Andersen and Petersen [1] (referred to as AP) using the super-efficiency model for ranking efficient observations is not very useful, but is more useful in outlier detection. In the case of existing of outliers for super-efficiency measurement, we can thus apply the AP procedure to detect and remove the outliers before using our one-stage approach.

3.3 A scaling strategy

Due to the assumption of positive data in the SBM model (i.e., $x_{ij} > 0$ and $y_{rj} > 0$), Tone [20] proposed an approach to deal with zero and negative data. In particular, if there are zero elements in input data, the corresponding slack variables s_i^- can be neglected. For zero elements in output data, the author classifies into two cases: (i) if the target DMU does not have a function to produce the output, the corresponding variables s_r^+ can be removed from the objective function, (ii) if the target DMU has a potential function to produce the output but does not utilise it, the zero output value can be replaced by a small positive number or one tenth of the minimum positive output value. The approach for zeros in output data can also be applied to deal with the negative output data.

Although the approach can deal with zero and negative data in the SBM models, its applicability to real world problems is not really efficient. For example, we consider two DMUs with the significant difference of negative output values (e.g., -1,000 and -10). When the approach is applied, these two DMUs obtain the same scaled output value. It means that they have the same contribution of relative efficiency score with respect to the output. This is not true in practice. Therefore, the difference of scaled output values may affect to the accuracy of relative efficiency measure of DMUs. In addition, inputs and outputs with large values may have more impact on the measure of relative efficiency than those with small values.

To overcome the disadvantages, we propose a new scaling strategy in which the obtained values of inputs and outputs are scalar in a range of 1-101. Let $X_i^{\min}$ and $X_i^{\max}$ be the minimum and maximum values of ith input, respectively. We denote the current input value and the scaled input value by X_i^{current} and X_i^{scale}, respectively. We can compute the scaled input value by

$$X_i^{\text{scale}} = \frac{\left(X_i^{\text{current}} - X_i^{\min}\right) 100}{X_i^{\max} - X_i^{min}} + 1. \tag{56}$$

Similarly, we can apply it for computing the scaled output values. We then obtain the scaled data set of inputs and outputs that include the impact of magnitude. Hence, the strategy is efficient to solve real world problems.

4 Numerical Experiments

In the section, we investigate the computational efficacy of measuring the efficiency scores of DMUs by our one-stage model. We evaluate the performance of the proposed model on several datasets in the literature and a case study. The obtained results are compared with those from other models, such as Tone [20, 21], Guo et al. [12]. All these models, including ones used to make a comparison, were implemented in Visual C++ and run on the same Microsoft Windows 7 Enterprise PC with an Intel Core i3-6100 Processor 2.30 GHz and 8 GB of RAM. The models were built and solved using the MILP solver of the IBM ILOG CPLEX version 12.4 callable library.

Table 2: A dataset of 5 DMUs (2 inputs, 2 outputs) in Tone [20].

DMU	x_1	x_2	y_1	y_2
A	4	3	2	3
B	6	3	2	3
C	8	1	6	2
D	8	1	6	1
E	2	4	1	4

Table 3: A dataset of 7 DMUs (2 inputs, 1 output) in Tone [21].

DMU	x_1	x_2	y_1
A	4	3	1
B	7	3	1
C	8	1	1
D	4	2	1
E	2	4	1
F	10	1	1
G	12	1	1

In the numerical experiments, the parameter values of our model were chosen as follows: $\underline{M} = 0.0001$ and $\overline{M} = 10,000$; while $\epsilon = 0.0001$ was used for the model of Guo et al. [12].

4.1 Benchmark datasets

Tables 2-4 present the datasets in the literature that are used to evaluate and compare our model with other models. In particular, they include the dataset of 5 DMUs (2 inputs, 2 outputs) in [20], the dataset of 7 DMUs (2 inputs, 1 output) in [21], and the dataset of 6 DMUs (4 inputs, 2 outputs) in [21].

We solved the datasets by the SBM model of Tone [20], the super SBM model of Tone [21], the one-stage model of Guo et al. [12] and our proposed model. Since our model is integrated based on the SBM and super SBM models, we first make a comparison with these two models to verify the accuracy of our model and linearisation technique. The obtained results are then compared with those of Guo et al. [12] to demonstrate the effectiveness of our proposed model. The comparison is based on both the solution quality and the computational time.

Tables 5 and 6 present the computational results for the dataset of 5 DMUs with 2 inputs and 2 outputs (see Table 2) solved by the models in 20, 21 and our model, respectively. The results show that our model can simultaneously obtain the efficiency scores of the inefficient DMUs and the super-efficiency scores of the efficient DMUs. As described in Section 3, if the SBM model is chosen to evaluate the target DMU

Table 4: A dataset of 6 DMUs (4 inputs, 2 outputs) in Tone [21].

DMU	x_1	x_2	x_3	x_4	y_1	y_2
D1	80	600	54	8	90	5
D2	65	200	97	1	58	1
D3	83	400	72	4	60	7
D4	40	1,000	75	7	80	10
D5	52	600	20	3	72	8
D6	94	700	36	5	96	6

Table 5: Results of the SBM model and the SupSBM model for the dataset of Table 2.

DMU	SBM					SupSBM				
	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	s_{2k}^{+*}	ρ_k^*	$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{y}_1^*$	$\tilde{y}_2^*$	δ_k^*
A	0	0.303	0.6061	0	0.798	4	3	2	3	1
B	0	0.4091	1.455	0	0.5682	6	3	2	3	1
C	0	0	0	0	1	10.67	1.333	8	1.333	1.333
D	0	0	0	0.6667	0.6667	8	1	6	1	1
E	0	0	0	0	1	2.909	5.818	1.455	2.182	1.455

Table 6: Results of our one-stage model for the dataset of Table 2.

DMU	OneSupSBM								
	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	s_{2k}^{+*}	$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{y}_1^*$	$\tilde{y}_2^*$	θ_k^*
A	0	0.303	0.6061	0	-	-	-	-	0.798
B	0	0.4091	1.455	0	-	-	-	-	0.5682
C	-	-	-	-	10.67	1.333	8	1.333	1.333
D	0	0	0	0.6667	-	-	-	-	0.6667
E	-	-	-	-	2.909	5.818	1.455	2.182	1.455

(assuming that it is an inefficient DMU), the variable values corresponding in the SupSBM model are arbitrary. Otherwise, if the SupSBM model is chosen to evaluate the target DMU (assuming that it is an efficient DMU), the variables values corresponding in the SBM model are arbitrary. Hence, we do not present the arbitrary values of these variables in the result tables. In the tables, the scores and slacks are the same as those obtained by solving sequentially the SBM and super SBM models. It demonstrates the accuracy of our integrated model and linearisation technique.

We continue to solve the datasets of Tables 3 and 4 by these models, and present the computational results in Tables 7-8 and 9-10, respectively. Once again, we can see that the proposed model can obtain the same results as the models of Tone [20] and Tone [21]. In this paper, we do not discuss the projection results of the datasets since they are the same as in [20] and [21].

Next, we make a comparison among our proposed model, the two-stage approach (i.e., solving the SBM first and then the SupSBM, namely SBM-SupSBM) and the one-stage model of Guo et al. [12] in additive DEA on the benchmark datasets (see in Tables 11). The comparison is based on both the solution quality and the computational time in seconds. The comparison results show that our model can obtain the same efficiency scores of DMUs as the two-stage approach (thus the efficiency scores of SBM-SupSBM are not reported in the tables), but less computational time. As compared with the one-stage model of Guo et al. [12], our model

Table 7: Results of the SBM model and the SupSBM model for the dataset of Table 3.

DMU	SBM				SupSBM			
	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	ρ_k^*	$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{y}_1^*$	δ_k^*
A	0	1	0	0.8333	4	3	1	1
B	0.6667	0	0.3333	0.619	7	3	1	1
C	0	0	0	1	10	1	1	1.125
D	0	0	0	1	6	2	1	1.25
E	0	0	0	1	4	4	1	1.5
F	2	0	0	0.9	10	1	1	1
G	4	0	0	0.8333	12	1	1	1

Table 8: Results of our one-stage model for the dataset of Table 3.

DMU	OneSupSBM						
	s_{1k}^{-*}	s_{2k}^{-*}	s_{1k}^{+*}	$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{y}_1^*$	θ_k^*
A	0	1	0	-	-	-	0.8333
B	0.6667	0	0.3333	-	-	-	0.6190
C	-	-	-	10	1	1	1.125
D	-	-	-	6	2	1	1.25
E	-	-	-	4	4	1	1.5
F	2	0	0	-	-	-	0.9
G	4	0	0	-	-	-	0.8333

Table 9: Results of the SBM model and the SupSBM model for the dataset of Table 4.

DMU	SBM							SupSBM						
	s_{1k}^{-*}	s_{2k}^{-*}	s_{3k}^{-*}	s_{4k}^{-*}	s_{1k}^{+*}	s_{2k}^{+*}	ρ_k^*	$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{x}_3^*$	$\tilde{x}_4^*$	$\tilde{y}_1^*$	$\tilde{y}_2^*$	δ_k^*
D1	0	0	0	0	0	0	1	80	627.9	54	8	90	5	1.012
D2	0	0	0	0	0	0	1	91.95	282.9	137.2	1.415	33.95	1.415	1.415
D3	0	0	0	0	0	0	1	83	525	72	4	60	7	1.078
D4	0	0	0	0	0	0	1	65	1,000	75	7	80	10	1.156
D5	0	0	0	0	0	0	1	90.24	768	34.56	4.8	92.16	5.76	1.586
D6	0	0	0	0	0	0	1	94	755.5	36	5	96	6	1.020

Table 10: Results of our one-stage model for the dataset of Table 4.

DMU	OneSupSBM												
	s_{1k}^{-*}	s_{2k}^{-*}	s_{3k}^{-*}	s_{4k}^{-*}	s_{1k}^{+*}	s_{2k}^{+*}	$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{x}_3^*$	$\tilde{x}_4^*$	$\tilde{y}_1^*$	$\tilde{y}_2^*$	θ_k^*
D1	-	-	-	-	-	-	80	627.9	54	8	90	5	1.012
D2	-	-	-	-	-	-	91.95	282.9	137.2	1.415	33.95	1.415	1.415
D3	-	-	-	-	-	-	83	525	72	4	60	7	1.078
D4	-	-	-	-	-	-	65	1,000	75	7	80	10	1.156
D5	-	-	-	-	-	-	90.24	768	34.56	4.8	92.16	5.76	1.586
D6	-	-	-	-	-	-	94	755.5	36	5	96	6	1.020

Table 11: Comparison results of the SBM-SupSBM model, our model and that of Guo et al. [12] for the small-size datasets.

Dataset	DMU	Guo et al. [12]		OneSupSBM		SBM-SupSBM
		θ_k^*	Time (s)	θ_k^*	Time (s)	Time (s)
5 DMUs (2 inputs, 2 outputs)	A	0.8485	0.162	0.798	0.138	0.220
	B	0.7197		0.5682		
	C	1.333		1.333		
	D	0.6667		0.6667		
	E	1.455		1.455		
7 DMUs (2 inputs, 1 outputs)	A	0.875	0.123	0.8333	0.097	0.145
	B	0.619		0.619		
	C	1.143		1.125		
	D	1.25		1.25		
	E	2		1.5		
	F	0.9		0.9		
	G	0.8333		0.8333		
6 DMUs (4 inputs, 2 inputs)	D1	1.014	0.217	1.012	0.153	0.233
	D2	1.648		1.415		
	D3	1.192		1.078		
	D4	1.176		1.156		
	D5	1.732		1.586		
	D6	1.047		1.020		

achieves the efficiency scores of DMUs less than or equal to those of the one-stage model, which verifies the fact that SBM models produce the more precisely efficiency scores than the additive (slacks-based) DEA models. In addition, our computation time is less than that of Guo et al. [12]. All these show the effectiveness of our model as compared with other models for solving the benchmark datasets.

4.2 A case study

We describe a case study used to test the performance of our model and other models in practical applications. In the case study, DMUs are construction companies in Nottingham City, the United Kingdom. For DMUs, we consider the following inputs and outputs for efficiency evaluation in terms of financial performance indicator (see Figure 2):

- **Inputs:** total assets x_1 (thousand GBP), the number of employees x_2 (persons), working capital needs x_3 (thousand GBP), wages and salaries x_4 (thousand GBP).
- **Outputs:** profit/loss after taxation y_1 (thousand GBP), profit margin y_2 (%), credit score y_3 (0-100), turnover y_4 (thousand GBP), return on capital employed y_5 (%).

The evaluation of DMUs gives us a general overview of relative financial performance indicator of construction companies in Nottingham City in the United Kingdom. We can determine the set of inefficient construction companies and the relevant elements that cause their inefficiency. From that, these companies may focus on dealing with the reasons for the improvement of their efficiency. Appendix A shows the scaled inputs and outputs of DMUs in the case study.

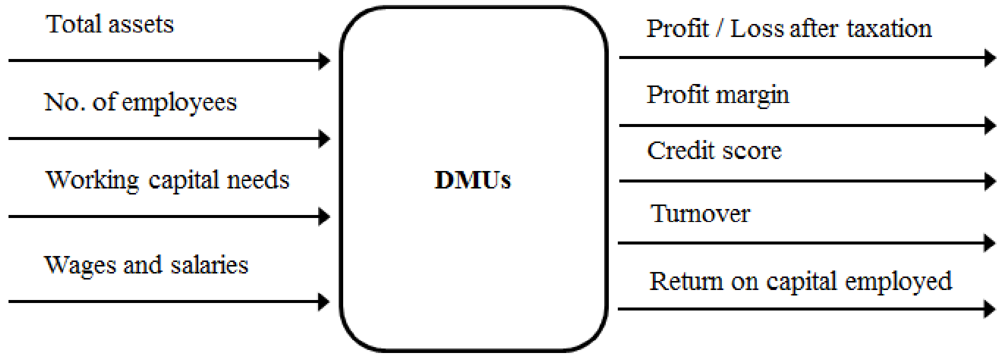

Figure 2: Inputs and outputs of DMUs in the case study.

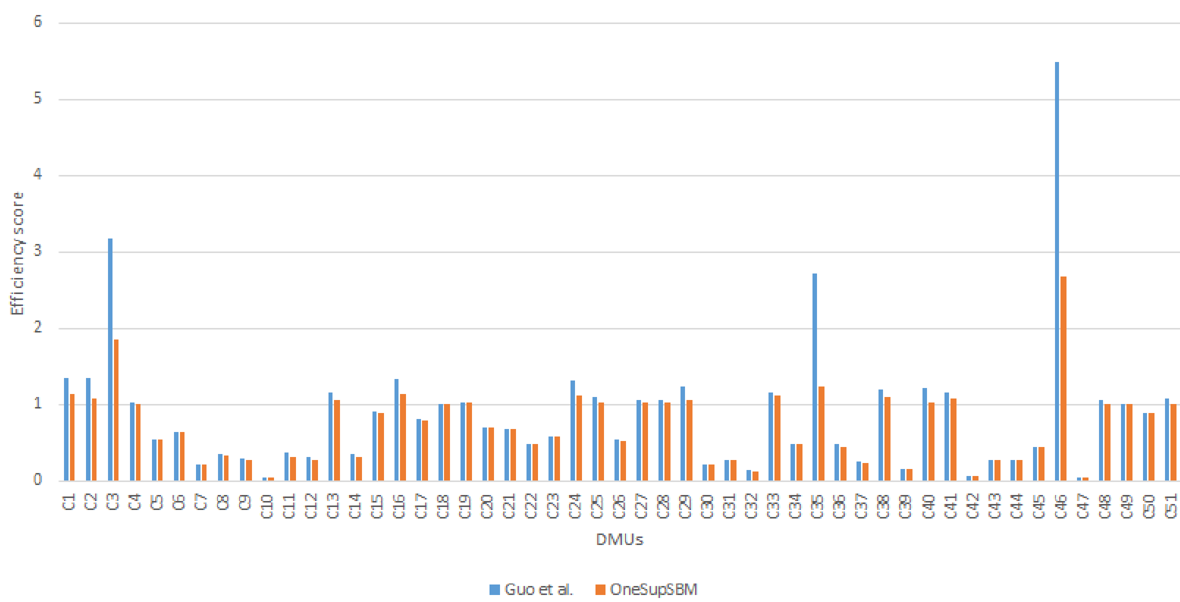

Figure 3: Comparison results of our model and that of Guo et al. [12] for the case study.

We continue to solve the case study by our model, the SBM-SupSBM model and that of Guo et al. [12]. Figure 3 shows that our model can achieve the efficiency scores of DMUs less than or equal to those of the model proposed by Guo et al. [12] (since the SBM-SupSBM model has the same efficiency scores of DMUs as our model, we do not report them in the figure). Once again, this verifies the fact that the efficiency scores obtained by SBM models are more precisely than those of the additive (slacks-based) DEA models. The computation time of our model (0.753 seconds) is faster than that of the SBM-SupSBM model (1.148 seconds) and that of Guo et al. [12] (1.404 seconds), which demonstrates the applicability of our model for solving large-size instances.

As considering the practical aspect for the case study, it can be seen that 22 out of 51 construction companies (approximately 43.14%) in Nottingham City are evaluated to be efficient, in terms of the financial performance indicator, while 29 remaining companies (approximately 56.86%) are inefficient. The average score of the financial performance indicator for all the companies is 0.73 and the standard deviation is 0.49. In general, the figures show that many construction companies in Nottingham City might be operating less effectively, and there exists a significant difference between the groups of efficient and inefficient companies. Investigating top three companies with the lowest financial performance indicator (i.e., C10, C42 and C47), it can be seen that (i) C10 may significantly improve its score if it may increase the outputs (e.g., profit/loss after taxation and return on capital employed), (ii) C42 should cut working capital needs, and increase credit score to improve its score, and (iii) C47 must cut the number of employees, working capital needs, wages and salaries, and increase profit margin to improve its score.

5 Conclusions and Future Work

The traditional solution approaches in DEA require identification of the efficient DMUs before applying the super-efficiency DEA models for the DMUs to achieve their super-efficiency scores, and vice versa. Therefore, the approaches entail a relatively high computational cost to obtain the scores of all DMUs, especially in large-scale practical applications. Guo et al. [12] proposed the one-stage solution approach in which two efficiency and supper-efficiency measure models are integrated into a single model. However, this is an integrated additive (slacks-based) DEA model that requires a post-computation process to obtain the efficiency scores of DMUs. We have developed an integrated model of the SBM model of Tone [20] and the super SBM model of Tone [21]. Our objective function can directly obtain the efficiency and super-efficiency scores of DMUs without the post-computation process. We also construct a linearisation technique to deal with the resulted non-linear integrated model. In addition, a scaling strategy that includes impact of magnitude in inputs and outputs is developed to address the negative and zero cases of inputs and outputs in the practical applications. A case study, along with several examples in the literature, are constructed to evaluate the proposed model. The experimental results demonstrate the accuracy and the computation time effectiveness of our model as compared with other models. The idea of switching the SBM model and the super SBM model, along with the proposed linearisation technique, can be easily applied in other fields.

In the case study, our focus is exclusively on firms' financial functioning. However, we can include inputs/outputs relevant to environmental and social aspects (e.g., CO2 emission, waste management, etc.) for a more realistic application. In addition, since using uniform weights for inputs and outputs may be unrealistic, we should engage with stakeholders (e.g., city council) to obtain the appropriate weights by multi-criteria decision analysis. We may also enrich the methodology to represent firm's responses to policy measures.

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Appendix A: A case study of 51 DMUs (4 inputs, 5 outputs).

DMU	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5
C1	1.00	1.45	15.98	1.00	1.43	10.36	1.00	2.35	37.32
C2	2.33	1.22	17.29	1.57	1.00	1.00	97.61	1.72	17.98
C3	2.83	1.22	17.63	2.34	5.79	70.03	85.75	3.75	57.25
C4	4.82	3.24	16.74	3.70	2.34	41.00	101.00	1.00	20.88
C5	5.40	11.44	20.13	3.87	5.15	22.99	101.00	12.92	44.77
C6	5.58	8.18	17.47	6.51	5.70	30.58	101.00	10.32	46.99
C7	6.10	6.84	21.22	7.37	2.37	7.32	101.00	13.28	26.76
C8	6.37	16.38	19.34	10.40	4.63	19.12	101.00	12.39	32.01
C9	6.52	6.84	21.22	7.37	3.31	11.42	94.22	13.29	29.27
C10	7.45	9.53	19.84	8.49	7.02	14.30	82.36	18.91	1.00
C11	7.80	7.62	23.46	5.94	4.80	28.26	101.00	8.18	27.02
C12	8.06	6.16	19.78	6.80	3.30	13.08	94.22	11.65	25.51
C13	8.94	20.19	21.39	15.40	21.32	53.91	89.14	25.41	101.00
C14	8.96	3.58	19.33	3.99	3.04	16.46	101.00	7.95	22.32
C15	9.97	20.19	21.39	15.40	20.98	53.30	89.14	25.41	83.25
C16	10.49	7.40	11.30	8.52	2.68	3.66	97.61	38.93	26.60
C17	10.67	20.19	21.39	15.40	19.46	50.03	94.22	25.41	73.69
C18	10.91	7.06	19.76	6.07	19.74	100.17	101.00	12.31	53.93
C19	11.25	7.06	18.35	6.07	19.90	101.00	101.00	12.29	52.37
C20	11.94	3.81	18.72	3.66	10.54	46.43	90.83	14.45	34.70
C21	11.98	8.52	17.96	2.95	7.47	39.45	101.00	10.82	34.64
C22	12.53	7.06	13.23	5.74	4.46	9.98	97.61	24.64	36.63
C23	13.49	6.16	23.97	5.44	13.64	38.84	101.00	22.14	37.50
C24	13.94	3.24	11.90	4.00	11.73	33.30	85.75	22.62	42.23
C25	14.02	10.32	6.02	7.50	13.09	62.55	89.14	12.89	61.30
C26	14.77	24.91	26.84	20.38	24.60	72.80	94.22	20.07	57.12
C27	14.81	10.32	5.98	7.50	14.32	68.04	89.14	12.80	56.53
C28	15.13	10.32	20.49	13.74	25.73	70.81	101.00	23.69	70.42
C29	15.32	12.78	22.62	5.71	9.40	11.53	101.00	47.47	53.22
C30	16.32	8.07	20.58	7.20	3.56	8.15	101.00	22.24	22.58
C31	16.96	17.05	12.53	12.52	5.88	12.97	84.05	16.96	27.70
C32	17.98	14.36	21.22	12.61	2.34	6.26	94.22	25.27	19.63
C33	19.62	3.69	18.72	2.90	14.99	63.99	101.00	14.45	28.96
C34	20.76	50.49	33.44	29.87	35.37	82.50	94.22	31.38	70.81
C35	21.23	1.00	39.65	2.36	2.55	1.89	77.27	45.62	19.75
C36	24.78	15.59	24.23	10.35	20.75	88.81	101.00	14.60	33.04
C37	25.18	8.63	30.17	6.21	6.63	20.50	101.00	18.64	25.09
C38	28.34	3.92	9.32	5.36	9.82	16.73	89.14	36.17	33.52
C39	31.23	37.70	37.89	21.72	11.18	14.96	77.27	32.76	43.85
C40	31.78	5.71	17.59	6.30	2.26	1.22	89.14	56.72	18.39
C41	33.22	15.59	33.36	10.20	55.45	98.89	89.14	38.29	51.49
C42	41.65	3.69	66.88	4.80	10.17	47.98	4.39	12.58	20.86
C42	43.21	43.65	24.60	29.71	22.24	12.91	101.00	86.52	39.85
C43	43.21	43.65	24.60	29.71	22.24	12.91	101.00	86.52	39.85
C45	49.08	24.23	5.20	24.12	20.63	23.11	101.00	59.46	32.34
C46	49.43	12.56	1.00	14.24	28.33	29.70	101.00	64.45	45.07
C47	65.94	101.00	82.96	101.00	18.37	11.75	89.14	85.56	26.03
C48	80.69	8.18	65.25	7.98	37.80	55.68	94.22	46.08	42.42
C49	82.45	52.74	28.09	55.69	101.00	70.36	89.14	101.00	44.64
C50	85.16	52.74	28.09	55.69	101.00	70.36	94.22	101.00	43.33
C51	101.00	13.01	101.00	11.17	57.64	69.53	101.00	56.52	26.79